14/8/2020

B. Sc. Part II 4th Paper

DIFF. EQNs. (LDE CC) (Contd.)

Q. Solve $(D^2-2D+1)y = x^2 e^x$

Soln.

For CF

$D^2-2D+1=0 \Rightarrow (D-1)^2=0$

$\Rightarrow D=1,1$

$\therefore$ C.F. $= (c_1+c_2x)e^x$

Now, $PI = \frac{1}{(D-1)^2} x^2 e^x = \frac{1}{(D-1)^2} e^x x^2$

$= e^x \frac{1}{[(D+1)-1]^2} x^2 = e^x \cdot \frac{1}{D^2} x^2$

$= e^x \cdot \frac{1}{D} \int x^2 dx = e^x \cdot \frac{1}{D}\left(\frac{x^3}{3}\right)$

$= \frac{e^x}{3} \int x^3 dx = \frac{e^x}{3} \cdot \frac{x^4}{4}$

$\Rightarrow PI = \frac{x^4 e^x}{12}$

Now, the ~~gen~~ solution is given by

$y = CF + PI$

$\Rightarrow y = (c_1+c_2x)e^x + \frac{x^4 e^x}{12}$.

Q. Solve $(D^2+3D+2)y = e^{2x}.\sin x$

Soln.

for CF, $D^2+3D+2=0$

$\Rightarrow D^2+2D+D+2=0 \Rightarrow D(D+2)+1(D+2)=0$

$\Rightarrow (D+2)(D+1)=0 \Rightarrow D=-1,-2.$

$CF = Ae^{-x}+Be^{-2x}$ —— (1)

for PI

$$PI = \frac{1}{D^2+3D+2} e^{2x}.\sin x$$

$$\Rightarrow PI = e^{2x}. \frac{1}{(D+2)^2+3(D+2)+2} \sin x$$

$$\Rightarrow PI = e^{2x}. \frac{1}{D^2+7D+12} \sin x$$

$$\Rightarrow PI = e^{2x}. \frac{1}{-1^2+7D+12} \sin x = e^{2x}. \frac{1}{7D+11} \sin x$$

$$= e^{2x}. \frac{(7D-11)}{(7D+11)(7D-11)} \sin x$$

$$= e^{2x}. \frac{(7D-11)\sin x}{49D^2-121} = \frac{e^{2x}.(7D-11)\sin x}{49\times(-1^2)-121}$$

$$\Rightarrow PI = \frac{e^{2x}}{-170}\left[7D(\sin x)-11(\sin x)\right] = \frac{e^{2x}(7\cos x - 11\sin x)}{-170}$$ —— (2)

Hence, the complete soln is $y = CF + PI$ given by (1) and (2).